Nuclear Levels of Cd^{115} and Cd^{117} from (d,p) Reaction Studies*

R. J. SILVA AND G. E. GORDON†
Chemistry Division, Oak Ridge National Laboratory, Oak Ridge, Tennessee
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Levels of Cd^{115} and Cd^{117} below about 3 MeV have been studied by observation of the energy and angular distributions of protons emitted in the $Cd^{114}(d,p)$ and $Cd^{116}(d,p)$ reactions, respectively. For many of the levels, l values of the captured neutrons have been assigned by comparison of the angular distributions with calculations based on the distorted-wave Born approximation. Results are analyzed in terms of the strong-pairing model of the nucleus as treated by Kisslinger and Sorensen. The experimental results are in good general agreement with predictions of the model, with the major exception that the energy of the $g_{7/2}$ single-particle orbital is found to be lower than predicted in the cadmium region. The Q values for the (d,p) reactions to the ground states of Cd^{115} and Cd^{117} are 3.939 ± 0.020 and 3.550 ± 0.020 MeV, respectively. In both cases the ground states have spin and parity $\frac{1}{2}+$. The $\frac{11}{2}-$ isomers lie 173 ± 10 and 133 ± 10 keV above the ground states in Cd^{115} and Cd^{117}, respectively.

I. INTRODUCTION

By virtue of their good energy resolution, high-energy tandem Van de Graaff accelerators and semiconductor detectors have made it easy to excite and identify by charged-particle reactions large numbers of nuclear energy levels for a given nuclide. In particular, the (d,p) reaction is known to be a powerful tool in the study of nuclear levels.[1–3] Comparisons of measured angular distributions of protons from the (d,p) reaction with theoretical calculations based on the distorted-wave Born approximation (DWBA) have been quite successful in determining the orbital angular momentum of captured neutrons and hence in providing information on the spins and parities of nuclear states. In addition, these comparisons yield the spectroscopic factors, i.e., the fraction of the single-particle excitation strength for a given level.

Recently, Kisslinger and Sorensen have published the results of calculations of nuclear energy levels, spins and parities, and occupation numbers (a quantity directly related to the spectroscopic factor) for a number of odd-mass spherical nuclei in terms of a short-range pairing force between like nucleons plus a quadrupole-quadrupole interaction between all nucleons.[4] Besides the single-quasiparticle levels, others arising from the coupling of single-particle levels to vibrational modes of an even-even core are included, i.e., states of one quasiparticle with zero-, and one-, and two-phonon vibrational states. These published results include calculations for several cadmium isotopes. It seems worthwhile, insofar as is possible, to compare the present experimental (d,p) data for Cd^{114} and Cd^{116} with these calculations, as these comparisons not only lead to information about the nuclear states involved but also have been successfully used to test the model.[5–9]

II. EXPERIMENTAL PROCEDURE

Experiments were performed using a 12.000 ± 0.005-MeV deuteron beam from the Oak Ridge National Laboratory tandem Van de Graaff accelerator. Following magnetic analysis and collimation to $\frac{1}{8}$-in. diam by a set of tantalum collimators, the beam was introduced into a 12-in.-diam scattering chamber. After passing through the target, the beam was collected in a Faraday cup and the beam current integrated.

The Cd^{114} and Cd^{116} targets were in the form of metal foils prepared by cold rolling and were of thickness 1.35 and 1.45 mg/cm^2, respectively. The former was enriched to 98.4% in Cd^{114} and the latter to 94.1% in Cd^{116}.[10] Uncertainty in the target thickness was $\pm10\%$.

Energy spectra of emitted or scattered charged particles were obtained as a function of angle relative to the beam direction through the use of a movable, 2-mm-thick surface-barrier silicon detector made from lithium-ion drifted material. The detector, which was equipped with a tantalum collimator, subtended a solid angle of 4.11×10^{-4} sr. The pulse-amplification system used was of the low-noise, charge-sensitive type.[11] Final pulse-height analysis was carried out using a 400-channel transistorized pulse-height analyzer (PHA). The observed energy resolution for the entire system, including energy straggling in the targets, was measured as about 50 keV full width at half the maximum height of the proton peaks studied.

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† Oak Ridge Institute of Nuclear Studies Summer Participant. Permanent address: Department of Chemistry, Massachusetts Institute of Technology, Cambridge, Massachusetts.

[1] M. H. MacFarlane and J. B. French, Rev. Mod. Phys. **32**, 567 (1960).

[2] J. B. French, in *Nuclear Spectroscopy, Part B*, edited by F. Ajzenberg-Selove (Academic Press Inc., New York, 1960), Chap. V. G.

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[6] B. L. Cohen and R. E. Price, Phys. Rev. **121**, 1441 (1961).

[7] B. L. Cohen, R. H. Fulmer, and A. L. McCarthy, Phys. Rev. **126**, 698 (1962).

[8] R. H. Fulmer and A. L. McCarthy, Phys. Rev. **131**, 2133 (1963).

[9] Bibijana Cujec, Phys. Rev. **131**, 735 (1963).

[10] The foils and compositions were supplied by the Isotopes Division, Oak Ridge National Laboratory.

[11] J. L. Blankenship and E. Fairstein, U. S. Atomic Energy Commission, Office of Technical Information Service, Document TID-6119, August 1960 (unpublished).

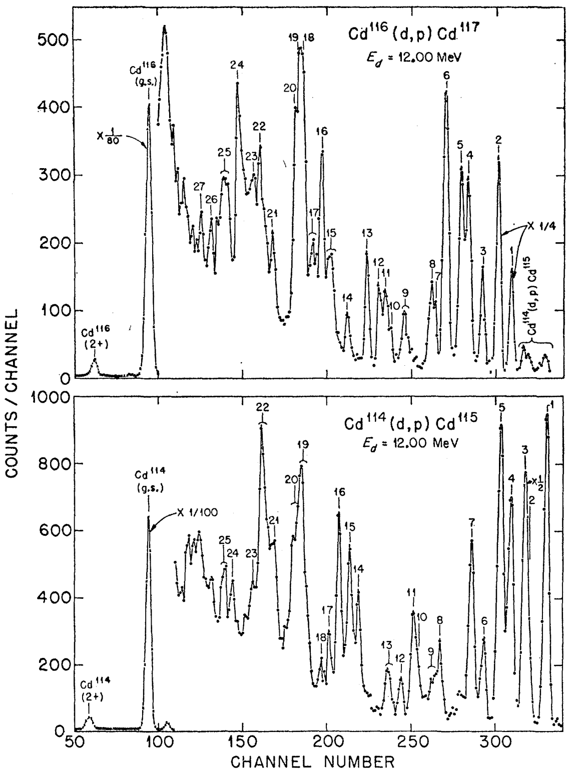

FIG. 1. Energy spectra of protons from (d,p) reactions on Cd^{114} and Cd^{116} obtained at a laboratory angle of 100°. Numbered groups were determined to be associated with energy levels of Cd^{115} and Cd^{117}. Also shown are the deuteron peaks associated with scattering from the ground and first excited states of Cd^{114} and Cd^{116}.

Absolute energies were assigned to proton peaks through the use of a PHA channel-number-versus-energy curve. This curve was obtained by recording the channel positions of peaks corresponding to elastic scattering of deuterons of precisely known beam energies ranging from 10.5 to 15 MeV in 0.5-MeV steps. A precision mercury-relay pulse generator, whose pulses were fed into the input of the preamplifier, was used to interpolate between the recorded energies. During the cadmium runs, the energy calibration was checked periodically by comparing the pulse generator with proton and deuteron peaks of known energies arising from reactions on cadmium and the impurities carbon and oxygen. The uncertainty in assigning absolute proton energies was determined to be ±20 keV.

Because of fluctuations and pulsing in the beam current, the dead time recorded by the PHA was in error. Therefore, dead-time corrections were made with the aid of a silicon detector permanently mounted at 90° to the beam direction. Pulses from the 90° counter were amplified and passed through a pulse-height selector which was gated on pulses produced by the elastically scattered deuterons. All these elastic events were recorded in one of two scaling units. The second unit recorded only those elastic events which occurred while the PHA storing unit was "busy." The latter was accomplished through the use of coincidence circuits operating on pulses from the 90° detector and the dead-time circuit of the analyzer. The two scaler readings were used to compute the true dead-time correction which was applied to the data obtained from the movable detector.

III. RESULTS

A. Determination of Energy Levels

Proton energy spectra were obtained for the (d,p) reactions on Cd^{114} and Cd^{116} as a function of laboratory angle from 25° to 160°. Figure 1 shows typical pulse-height spectra obtained at 100°. Proton groups associated with energy levels in Cd^{115} and Cd^{117} are numbered and were distinguished from those of other nuclei

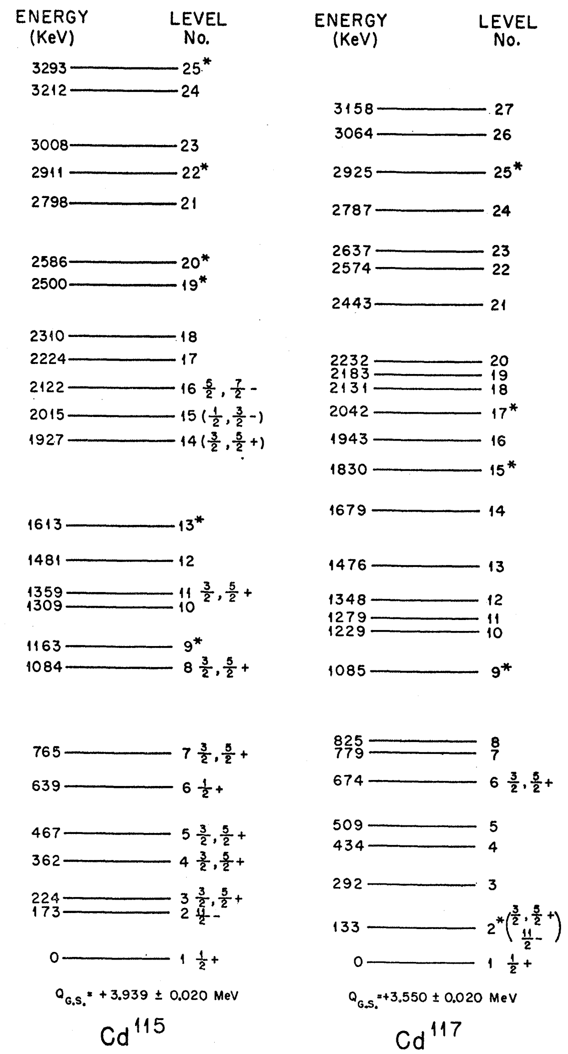

FIG. 2. Energy levels of Cd^{115} and Cd^{117} corresponding to numbered proton peaks of Fig. 1. Asterisks denote unresolved multiple levels. For single levels, the uncertainty in the energy relative to the ground state is ±10 keV. Spin and parity assignments were made from a comparison of experimental angular distributions with DWBA calculations.

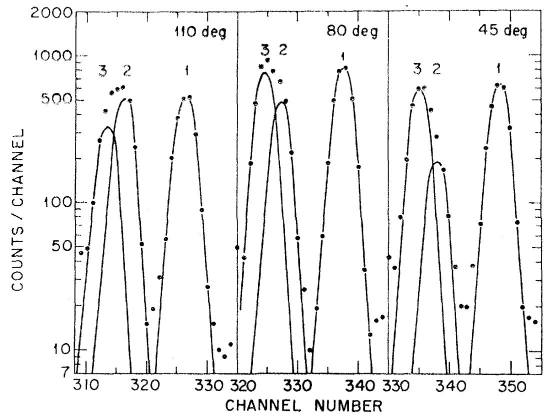

FIG. 3. Examples of partial separation of proton groups 2 and 3 of Cd^{115}.

(impurities) in the target by a careful study of the characteristic change in energy with laboratory angle for spectra taken at 50°, 75°, and 100°. In some cases, e.g., group 10 of Cd^{115}, the cross sections for production happen to be quite small at 100°, but the peaks appear clearly at other angles. A number of groups were checked at other angles and the relative energy assignments were determined to be good to ± 10 keV. Peaks that gave larger energy deviations or whose intensities were too small to be reliably assigned to reactions involving the principle cadmium isotope in the target were rejected.

For reactions leading to the ground states of Cd^{115} and Cd^{117}, the Q values were determined to be 3.939 ± 0.020 MeV and 3.550 ± 0.020 MeV, respectively. The energies of excitation above the ground state were obtained for the levels associated with the other numbered proton groups and are summarized in Fig. 2. The spin and parity assignments indicated will be discussed below. In some cases, indicated by an asterisk, the proton groups had widths greater than 50 keV, but could not be clearly resolved into more than one group. It is probable that these peaks consist of more than one component. The only levels previously known for these nuclides were levels 1, 2, 3, and 4 in Cd^{115}.[12,13] As levels 2 and 3 in Cd^{115} are only about 50 keV apart in energy, it was difficult to clearly distinguish the corresponding two separate proton peaks. However, the relative intensity of excitation of these two levels changed markedly with the angle of observations; thus, it was possible to determine the existence of the two levels and to make a rough separation of two proton groups (see Fig. 3). Although group 2 of Cd^{117} had a width of about 50 keV, experimental evidence will be presented below which indicates it results from the excitation of two closely spaced levels.

[12] J. M. Alexander, U. Schindewolf, and C. D. Coryell, Phys. Rev. **111**, 228 (1958).

[13] *Nuclear Data Sheets*, compiled by K. Way *et al.* (Printing and Publishing Office, National Academy of Sciences–National Research Council, Washington 25, D. C.), NRC 60-03-104.

B. Characterization of Energy Levels

Angular distributions are shown in Figs. 4 and 5 for numbered proton groups which were sufficiently well separated from neighboring peaks and extended enough above background to make extraction reliable. The errors shown are relative and arise primarily from the uncertainty in extraction of data from the experimental energy spectra.

In order to obtain information about the characteristic angular-momentum transfer $l_n\hbar$ and spectroscopic factor S involved in the excitation of a given level,[1–3] comparisons between the experimental angular distributions and distorted-wave Born approximation (DWBA) calculations were carried out. The DWBA calculations predict the "outgoing" proton intensities $(d\sigma_l/d\Omega)_{sp}$, for the excitation of a single-particle level as a function of angle, l_n and Q value.[14–16] These calcu-

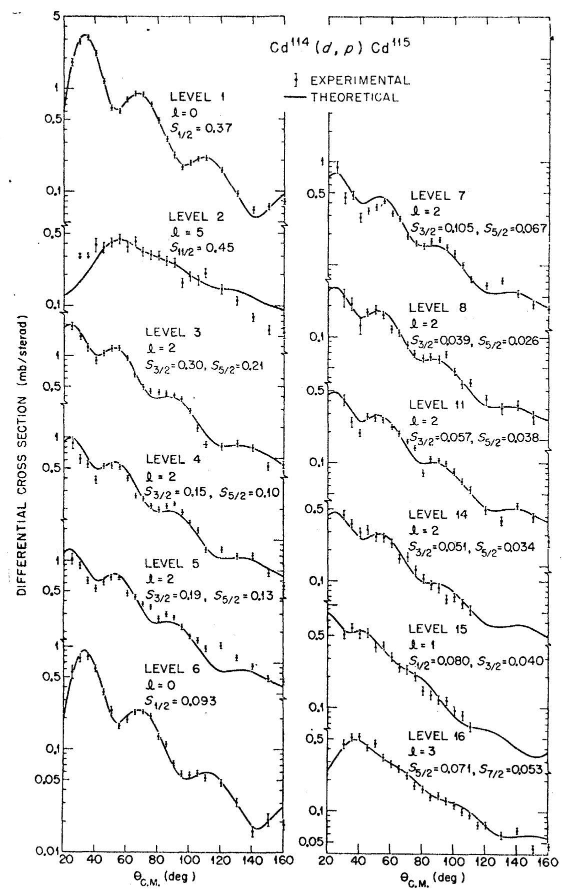

FIG. 4. Angular distributions of proton groups from Cd^{115} that could be reliably extracted from data. Solid curves are the results of DWBA calculations for the indicated angular-momentum transfer (l_n). Spectroscopic factors, S_j, shown were obtained from normalization of experimental data to theoretical curves.

[14] W. Tobocman, Phys. Rev. **115**, 99 (1959).

[15] N. Austern, in *Fast Neutron Physics, II*, edited by J. B. Marion and J. L. Fowler (Interscience Publishers, Inc., New York, 1962), Chap. V. D.

[16] W. Tobocman, *Theory of Direct Nuclear Reactions* (Oxford University Press, New York, 1961).

lations were carried out for $l_n=0, 1, 2, 3, 4, 5$ on the Oak Ridge IBM-7090 computer using a program developed by Bassel *et al.*[17] For these calculations, one needs the parameters obtained from optical-model fitting to experimentally observed deuteron and proton-elastic scattering at the same energy as the reaction studied. The optical-model potential used was of the form

$$U(r)=-V(e^x+1)^{-1}+i4W(d/dx')(e^{x'}+1)^{-1},$$
$$x=(r-r_{0s}A^{1/3}/a_s),\quad x'=(r-r_{0I}A^{1/3}/a_I).$$

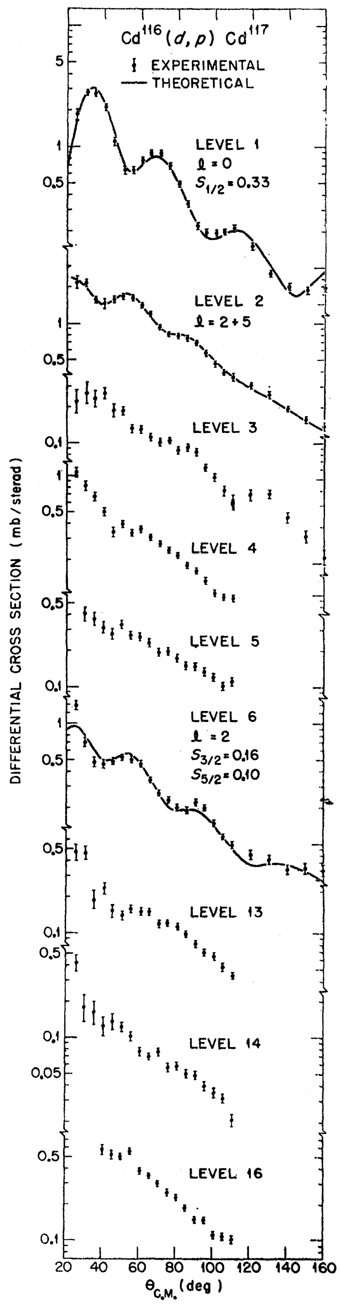

FIG. 5. Angular distributions of proton groups from Cd^{117}. The curve compared with level 2 of Cd^{117} is of the precise shape and magnitude as the sum of the angular distributions of levels 2 and 3 of Cd^{115} (see text).

[17] R. H. Bassel, R. M. Drisko, and G. R. Satchler, Oak Ridge National Laboratory Report, ORNL-3240, 1962 (unpublished).

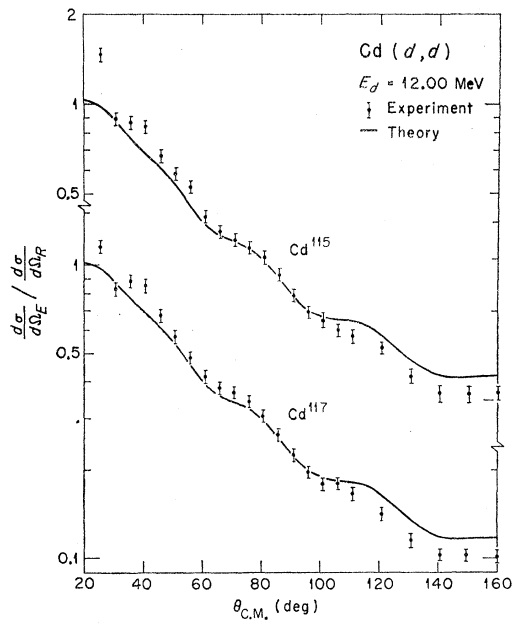

FIG. 6. Differential cross sections obtained for elastic scattering of 12-MeV deuterons from Cd^{114} and Cd^{116} (plotted as ratio to Rutherford scattering). The solid curves are cross sections predicted by the optical-model potential used in the DWBA calculations. Parameters are given in the text.

In addition, a Coulomb potential based on the assumption of a uniformly charged sphere was used.[14] The parameters ($V=77$ MeV, $W=17$ MeV, $r_{0s}=1.07$ F, $a_s=0.894$ F, $r_{0I}=1.28$ F, $a_I=0.702$ F) used to describe the deuteron elastic scattering were obtained from an analysis by Perey and Perey of data obtained from the scattering of 11.8-MeV deuterons by cadmium.[18] These parameters give a reasonably good account of the elastic scattering data obtained from this experiment as shown in Fig. 6. The parameters ($V=53$ MeV, $W=16$ MeV, $r_{0s}=1.25$ F, $a_s=0.65$ F, $r_{0I}=1.25$ F, $a_I=0.47$ F) for proton elastic scattering were obtained from the analysis of data from 22.2-MeV protons on cadmium given by F. Perey.[19] Corrections were made to V and W for the lower proton energy involved in this work using the equations given in Ref. 19. In these calculations, spin-orbit coupling effects on the elastic scattering were neglected and the "zero-range" approximation was made.[20] The wave function of the bound-state neutron in the residual nucleus was calculated using a real Saxon potential of radius 1.25 F and diffuseness 0.65 F. The depth was adjusted to give a binding energy for the neutron which corresponds to the observed Q values for the reactions.

A comparison with the theoretical curves was sufficient to assign unique l_n values for a number of the experimental angular distributions as shown in Figs. 4

[18] C. M. Perey and F. G. Perey, Phys. Rev. **132**, 755 (1963).
[19] F. G. Perey, Phys. Rev. **131**, 745 (1963).
[20] N. Austern, R. M. Drisko, E. C. Halbert, and G. R. Satchler, Phys. Rev. **133**, B3 (1964).

TABLE I. Comparison of experimental data for $Cd^{114}(d,p)Cd^{115}$ with pairing-model predictions.

Neutron orbital	Levels taken	Pairing theory[a] ϵ_j (MeV)	E_j(rel) (MeV)	U_j^2	n_j	Experimental E_j(rel) (MeV)	U_j^2 (ΣS_j)	n_j
$s_{1/2}$	1,6	1.310	0	0.35	1.30	0	0.45	1.08
$d_{3/2}$	3,4,5,8	2.864	0.23	0.83	0.69	0.26	0.68	1.28
$d_{5/2}$	7,11	−0.016	0.88	0.10	5.38	0.85	0.11	5.34
$g_{7/2}$	...	1.113	0.09	0.29	5.70	>0.52[b]	<0.15	>6.80
$h_{11/2}$	2	2.519	0.05	0.76	2.91	0.045	0.45	6.60
$\Sigma_j n_j$					16			21.1

[a] Calculated according to Ref. 4 with parameters for Cd^{114}: $\lambda=1.73$, $\Delta=1.31$ MeV; Cd^{115}: $\lambda=1.86$, $\Delta=1.30$ MeV. ϵ_j and E_j refer to Cd^{115}, U_j^2 and n_j to Cd^{114}.
[b] Calculated from experimental ΣS_j.

and 5. As is often the case, the best fits were obtained by excluding reactions within the nuclear volume. This was accomplished by using a lower "cutoff" radius (6.7 F in this case) in the radial integrals. However, this procedure was not necessary to distinguish between and assign the different l_n values. Since the ground-state spin and parity of the target nucleus is 0+, the usual angular momentum and parity relationships limit the final state spin, J_f, to $l_n\pm\frac{1}{2}$ and allow parity change only for odd l_n. This leads to an ambiguity in spin assignment, except for $l_n=0$, as shown in Fig. 2. The spectroscopic factors, S_j, given in Figs. 4 and 5 were obtained from the relationship

$$[d\sigma_j/d\Omega]_{\exp}=[d\sigma_l/d\Omega]_{\rm sp}(2J_f+1)S_j.$$

The angular distributions of levels 14 and 15 in Cd^{115} could be fit nearly equally as well with l_n of 1 or 2. The assignments in parentheses result from what was considered to be the better of the two fits. Bahn, Pate, Fink, and Coryell have studied the levels of Cd^{115} by β- and γ-ray spectroscopy of radiations emitted in the decay of Ag^{115}.[21] Through the use of β- and γ-decay selection rules and observed transition intensities, they have been able to assign spin values to many of the levels observed in this work. The consistency between their spin assignments and our l_n-value determinations is quite good, with the following exceptions. For level 15, their spin assignment would lead to $l_n>1$ in our work. Although the angular distribution is fairly well fitted by an $l_n=2$ calculated curve, $l_n=1$ gives a slightly better fit. For level 16, Bahn *et al.* prefer $l_n=1$. Although the angular distribution is rather structureless and would fit the calculated $l_n=1$ moderately well, $l_n=3$ gives a somewhat better fit. The spins of the well-known isomers[12,13] of Cd^{115} have recently been directly measured as $\frac{1}{2}$ and $\frac{11}{2}$ in agreement with our assignments for levels 1 and 2.[22]

As Cd^{117} is also known to have isomers with half-lives of approximately 3.2 and 2.7 h,[23] we expected to find a low-lying state whose angular distribution was characteristic of $l_n=5$, analogous to level 2 of Cd^{115}. The ground state of Cd^{117} was determined to be $\frac{1}{2}+$, but a search of the other low-lying proton groups failed to yield an $l_n=5$ transition. However, level 2 of Cd^{117}, though strongly excited, exhibits an anomalous angular distribution. In fact, the distribution has precisely the same shape and magnitude as that obtained for the sum of levels 2 and 3 of Cd^{115}. The solid curve compared with the data for level 2 of Cd^{117} in Fig. 5 is the best shape for the sum of levels 2 and 3 of Cd^{115}. Therefore, group 2 of Cd^{117} is thought to arise from the excitation of two levels ($\frac{11}{2}-$ and $\frac{3}{2}$ or $\frac{5}{2}+$) spaced less than 20 keV apart as determined from the peak width. Except for levels 1, 2, and 6, all other levels of Cd^{117} are, in general, excited with approximately one fourth the intensity of the levels in Cd^{115} of about the same energy. This made the analysis of the experimental data less reliable for the Cd^{117} levels and also indicates a loss in single-particle character. Both these facts tend to "wash out" the characteristic oscillations predicted by the DWBA calculations. Therefore, as seen in Fig. 5, many of the angular distributions for Cd^{117} were rather structureless, and it was not possible to assign l_n values for any of these levels with certainty.

IV. DISCUSSION

The neutron subshells filling in this region are $s_{1/2}$, $d_{3/2}$, $d_{5/2}$, $g_{7/2}$, and $h_{11/2}$. In the pairing-theory approximation, the subshells do not fill stepwise as they would in the absence of residual interactions, but all orbits of a major shell fill simultaneously and, therefore, are partially occupied. The occupancy V_j^2 gives the fraction by which the jth orbitals are filled; U_j^2 is the fraction empty. The energy levels excited in the (d,p) reaction would be the single-quasiparticle levels with energies E_j as given below.[24] However, the long-range quadrupole interaction leads to vibrational states and, by coupling phonons with single-quasiparticles, splits a given single-quasiparticle level into more than one

[21] E. L. Bahn, B. D. Pate, R. D. Fink, and C. D. Coryell, Phys. Rev. 136, B603 (1964).
[22] M. N. McDermott, R. Novick, B. W. Perry, and E. B. Saloman, Phys. Rev. 134, B25 (1964).
[23] C. W. Tang, thesis, Department of Chemistry, Massachusetts Institute of Technology, Sept. 1963 (unpublished); A. H. Wapstra and G. Chilosi, Instituut voor Kernphysisch Onderzoek, Amsterdam, 1963 (private communication).
[24] B. L. Cohen, R. H. Fulmer, A. L. McCarthy, and P. Mukherjee, Rev. Mod. Phys. 35, 332 (1963).

level. Reference 4 lists the energy levels, with spins and parities, to be expected from the model for Cd^{115}. Also listed are the parameters ϵ_j (the single-particle level energies), λ (the chemical potential) and Δ (half of the "energy gap") from which the E_j's and U_j^2's can be calculated using the equations

$$E_j=[(\epsilon_j-\lambda)^2+\Delta^2]^{1/2} \quad (1)$$

and

$$U_j^2=\tfrac{1}{2}[1+(\epsilon_j-\lambda/E_j)]. \quad (2)$$

The predicted ϵ_j's and E_j's (the latter relative to the lowest energy or ground state, $E_{1/2}$) for Cd^{115} and the U_j^2's for Cd^{114} are listed in columns 3 to 5 of Table I. Also included are the numbers of neutrons in each subshell obtained from the relationship

$$n_j=(2j+1)V_j^2,$$

where $\sum_j n_j=16$ for Cd^{114}.

In order to compare the experimental data with the pairing-theory calculations, we have removed the ambiguities in $j=l\pm\frac{1}{2}$ by use of the spin assignments made by Bahn *et al.*[21] The numbered levels of Cd^{115} assigned to the various neutron orbitals are given in Table I. As the experimental data for Cd^{117} are very incomplete, no comparisons are made for this isotope.

The experimentally determined level scheme for Cd^{115} appears to be in fair qualitative agreement with that predicted by Kisslinger and Sorensen in that we observe levels of the predicted spins and parities at low energies; however, there is quantitative disagreement as far as actual energies are concerned. Fortunately, there are sum rules which enable one to determine the unperturbed single-quasiparticle energies and vacancies from the observed energy levels and spectroscopic factors.

The experimental relative E_j's can be obtained from the "centers of gravity" of the observed levels E_i of a given j from the relationship[5–9]

$$E_{cg}=\sum_i E_iS_i/\sum_i S_i,$$

which has been shown to be a good approximation to the more exact relationship given by Yoshida[25,26] and

$$E_j(\mathrm{rel})=E_{cg}(j)-E_{cg}(s_{1/2}).$$

The accuracy, of course, depends on all or nearly all levels of a given j being observed. A comparison of the relative E_j's obtained from pairing theory and the experiment are shown in Table I.

It has been shown for even-even isotope targets that $\sum_i S_i$ for all levels of same spin and parity is equal to U_j^2 of the pairing theory for the target nucleus.[25,26] One can compare the U_j^2's with $\sum S_j$ in Table I. A search of the proton spectra corresponding to excitations up to about 2 MeV failed to yield any $l=4$ transitions which would be expected for the $g_{7/2}$ levels if $U_{7/2}^2$ were as large as the predicted 0.29. From a consideration of the small unassigned peaks in the spectra, a fairly conservative limit of $\sum S_{7/2}<0.15$ was made.

Using the values of $\sum S_j$ for the $g_{7/2}$ and $s_{1/2}$ levels, together with the value of $\Delta=1.3$ used in Ref. 4, the relative $E_{7/2}$ was calculated from the relationship

$$E_{7/2}(\mathrm{rel})=\frac{\Delta}{2[V_{7/2}^2U_{7/2}^2]_{\exp}^{1/2}}-\frac{\Delta}{2[V_{1/2}^2U_{1/2}^2]_{\exp}^{1/2}}, \quad (3)$$

which can be derived from Eqs. (1) and (2).

The agreement between experiment and theory is about as good as one might expect from the errors inherent in the DWBA analysis and the pairing-model calculations except for the $h_{11/2}$ and the $g_{7/2}$ orbits. The experimental $\sum_j n_j$ is considerably too large. The most likely reason for this occurrence is the $\sum S_{11/2}$ being too small. The $h_{11/2}$ orbital shows the largest deviation of $\sum S_j$ from U_j^2. This is quite possible as only one $\frac{11}{2}-$ state was observed experimentally, whereas more than one is predicted. Also, there is a large uncertainty in $S_{11/2}$ due to the difficulty in extracting the data for this level from the energy spectra. The reasonable values of $\sum S_j$ for the well-characterized orbitals in this work would suggest that the DWBA cross sections are of approximately correct magnitude.

A lowering of the $g_{7/2}$ single-particle energy $\epsilon_{7/2}$ due to an extra stabilization of the g orbitals has been attributed to the interaction between the $g_{9/2}$-proton and $g_{7/2}$-neutron orbitals filling simultaneously in this mass region.[27] Kisslinger and Sorensen have attempted to take this effect into account with a Z-dependent correction to $\epsilon_{7/2}$. However, a comparison with the experimental data indicates that the single-quasiparticle energy for the $g_{7/2}$ orbital calculated from the parameters of Ref. 4 is too small, i.e., the single-particle energy is too large. A value more consistent with the experimental data is $\epsilon_{7/2}<0.4$ MeV relative to $\epsilon_{5/2}$. Cujec has found a similar trend for the $g_{7/2}$ orbital in the palladium region.[9]

Not much can be said about the levels in Cd^{117}. As was mentioned above, except for what appears to be the lowest energy member of a given j group, all the levels in Cd^{117} were excited with only about one-fourth the intensity of the levels in Cd^{115} of about the same energy. This is seen most dramatically in the $\frac{3}{2}+$ states. This effect could be due to a sharp drop in U_j^2's going from Cd^{115} to Cd^{117}; however, this would be contrary to the rather smooth variation in the U_j^2's predicted by the model. A similar effect is seen in the tin isotopes.[6] It is curious that this effect occurs at the closing of the $d_{3/2}$ shell-model orbital if the subshells fill stepwise in the sequence $g_{7/2}$, $d_{5/2}$, $d_{3/2}$ suggested by Mayer and Jensen.[28]

[25] S. Yoshida, Phys. Rev. **123**, 2122 (1961).
[26] S. Yoshida, Nucl. Phys. **38**, 380 (1962).
[27] B. L. Cohen, Phys. Rev. **127**, 597 (1962).
[28] M. G. Mayer and J. H. D. Jensen, *Elementary Theory of Nuclear Shell Structure* (John Wiley & Sons, Inc., New York, 1955), p. 58.

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Neutron-Neutron Scattering at Low Energies

MICHAEL J. MORAVCSIK
Lawrence Radiation Laboratory, University of California, Livermore, California
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This paper investigates the information contained in a neutron-neutron scattering experiment at low energies which could be performed by colliding beams coming from an underground nuclear explosion. The significance of such an experiment is discussed from the point of view of a check on charge symmetry and charge independence, and it is found that because of the electromagnetic complications in proton-proton scattering, and because of the proton-neutron mass difference, the knowledge of neutron-neutron scattering would be of considerable value. The functional form of the experimental data which is most convenient for analysis and the approximate relative magnitude of the terms is investigated, and it is concluded that for the kind of experiment which is envisaged (measuring cross sections to 10% from 20 keV to 2 MeV) only two parameters should be kept in the effective-range expansion. The connection between the number and distribution of energies at which the cross section is measured and the error on the individual measurements, on the one hand, and the accuracy of the effective-range parameters deduced from the experiments, on the other, is given explicitly and is found also to depend on the absolute magnitude of the scattering length. The results show that ten 10% measurements, suitably distributed between 20 keV and 2 MeV, can determine the sign of the scattering length to four standard deviations, the magnitude of the effective range to 50–70%, and the magnitude of the scattering length to about 3%. Finally, the relationship between the variation of the effective-range parameters and the corresponding variation in the parameters of the scattering potential is studied, and it is found that, while this relationship is strongly shape-dependent, a small change in the potential parameters, in any case, results in a large change in the scattering length, but a small one in the effective range. Numerical relationships show that, even in the worst case, the variation in the scattering length is about eight times the variation in the potential parameter. It is concluded that a 10% experiment at 20 energies between 20 keV and 2 MeV would be able to get information on the potential parameters sufficiently accurately so that charge-dependent or charge-symmetry violating effects could be detected.

I. INTRODUCTION

THIS paper was prompted by considerations of the possibility of measuring neutron-neutron scattering at low energies in a colliding beam experiment utilizing a single underground nuclear explosion.[1] Experimental aspects of this problem will not be discussed here, but it appears that it may be possible to measure this scattering cross section to an accuracy as high as 10%, from about 20 keV to about 2 MeV. The questions under investigation are: (a) why the knowledge of low-energy neutron-neutron scattering would be of interest (Sec. I); (b) the relationship of experimental data to the effective-range parameters to be determined (Sec. II); (c) the dependence of the uncertainty in the effective-range parameters on the number, distribution, and error of the experimental data points (Sec. III); and (d) the relationship between the errors on the effective-range parameters and the uncertainty in the parameters describing the scattering potential (Sec. IV). The conclusions are stated in Sec. V.

A precision knowledge of the neutron-neutron scattering parameters at low energy would be of interest for several reasons. Firstly, it would furnish a test of charge symmetry. Although charge symmetry is rather firmly believed, the substantial evidence for it comes exclusively from nuclear structure. Since our knowledge of the relationship of nuclear structure to the nuclear two-body problem is far from complete, there is much to be said for a direct check of charge symmetry using the two-nucleon interaction itself. Furthermore, there are several phenomena which will cause an apparent

[1] Charles D. Bowman and William C. Dickinson, University of California, Lawrence Radiation Laboratory Report No. UCRL-7859, 1964 (unpublished).